OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE **4725/01**

MATHEMATICS

Further Pure Mathematics 1

FRIDAY 11 JANUARY 2008 Morning

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

[Turn over

1 The transformation S is a shear with the y-axis invariant (i.e. a shear parallel to the y-axis). It is given that the image of the point (1, 1) is the point (1, 0).

(i) Draw a diagram showing the image of the unit square under the transformation S. [2]

(ii) Write down the matrix that represents S. [2]

2 Given that $\sum_{r=1}^{n}(ar^2 + b) \equiv n(2n^2 + 3n - 2)$, find the values of the constants a and b. [5]

3 The cubic equation $2x^3 - 3x^2 + 24x + 7 = 0$ has roots α, β and γ.

(i) Use the substitution $x = \frac{1}{u}$ to find a cubic equation in u with integer coefficients. [2]

(ii) Hence, or otherwise, find the value of $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}$. [2]

4 The complex number $3 - 4\mathrm{i}$ is denoted by z. Giving your answers in the form $x + \mathrm{i}y$, and showing clearly how you obtain them, find

(i) $2z + 5z^*$, [2]

(ii) $(z - \mathrm{i})^2$, [3]

(iii) $\frac{3}{z}$. [3]

5 The matrices **A**, **B** and **C** are given by $\mathbf{A} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix}$ and $\mathbf{C} = (2 \quad 4 \quad -1)$. Find

(i) $\mathbf{A} - 4\mathbf{B}$, [2]

(ii) **BC**, [4]

(iii) **CA**. [2]

6 The loci C_1 and C_2 are given by

$$|z| = |z - 4\mathrm{i}| \qquad \text{and} \qquad \arg z = \tfrac{1}{6}\pi$$

respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. [5]

(ii) Hence find, in the form $x + \mathrm{i}y$, the complex number represented by the point of intersection of C_1 and C_2. [3]

7 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} a & 3 \\ -2 & 1 \end{pmatrix}$.

(i) Given that **A** is singular, find a. [2]

(ii) Given instead that **A** is non-singular, find $\mathbf{A}^{-1}$ and hence solve the simultaneous equations

$$\begin{aligned} ax + 3y &= 1, \\ -2x + y &= -1. \end{aligned}$$ [5]

8 The sequence $u_1, u_2, u_3, \ldots$ is defined by $u_1 = 1$ and $u_{n+1} = u_n + 2n + 1$.

(i) Show that $u_4 = 16$. [2]

(ii) Hence suggest an expression for u_n. [1]

(iii) Use induction to prove that your answer to part **(ii)** is correct. [4]

9 **(i)** Show that $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$. [2]

(ii) The quadratic equation $x^2 - 5x + 7 = 0$ has roots α and β. Find a quadratic equation with roots α^3 and β^3. [6]

10 **(i)** Show that $\frac{2}{r} - \frac{1}{r+1} - \frac{1}{r+2} = \frac{3r+4}{r(r+1)(r+2)}$. [2]

(ii) Hence find an expression, in terms of n, for

$$\sum_{r=1}^{n} \frac{3r+4}{r(r+1)(r+2)}.$$ [6]

(iii) Hence write down the value of $\sum_{r=1}^{\infty} \frac{3r+4}{r(r+1)(r+2)}$. [1]

(iv) Given that $\sum_{r=N+1}^{\infty} \frac{3r+4}{r(r+1)(r+2)} = \frac{7}{10}$, find the value of N. [4]

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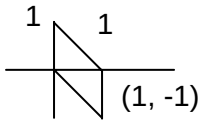
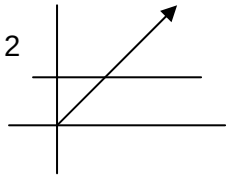
1	(i)  (ii) $\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$	M1 A1 B1 B1	 2 2 **4**	For 2 other correct vertices seen, correct direction of shear seen For completely correct diagram, must include scales Each column correct
2	$\frac{a}{6}n(n+1)(2n+1)+bn$ $a = 6$ $b = -3$	M1 A1 M1 A1 A1	 5 **5**	Consider sum as two separate parts Correct answer a.e.f. Compare co-efficients Obtain correct answers
3	(i) $7u^3 + 24u^2 - 3u + 2 = 0$ (ii) *EITHER* correct value is $-\frac{3}{7}$ *OR* correct value is $-\frac{3}{7}$	M1 A1 M1 A1ft M1 A1	 2 2 **4**	Use given substitution Obtain correct equation a.e.f. Required expression related to new cubic Their c / their a Use $\frac{\alpha+\beta+\gamma}{\alpha\beta\gamma}$ or equivalent Obtain correct answer
4	(i) $z^* = 3 + 4i$ $21 + 12i$ (ii) $3 - 5i$ $-16 - 30i$ (iii) $\frac{9}{25} + \frac{12}{25}i$	B1 B1 B1 B1ft B1ft M1 A1 A1	 2 3 3 **8**	Conjugate seen or implied Obtain correct answer Correct $z - i$ or expansion of $(z - I)^2$ seen Real part correct Imaginary part correct Multiply by conjugate Numerator correct Denominator correct
5	(i) $\begin{pmatrix} -13 \\ 1 \\ -10 \end{pmatrix}$ (ii) $\begin{pmatrix} 8 & 16 & -4 \\ 0 & 0 & 0 \\ 6 & 12 & -3 \end{pmatrix}$ (iii) (8)	B1 B1 M1 A1A1A1 M1 A1	 2 4 2 **8**	4**B** seen or implied or 2 elements correct Obtain correct answer Obtain a 3 x 3 matrix Each row (or column) correct Obtain a single value Obtain correct answer, must have matrix

6	(i)  (ii) $2\sqrt{3} + 2\,\mathrm{i}$	B1 B1 B1 B1 B1 B1 M1 A1	 5 3 **8**	Horizontal straight line in 2 quadrants Through (0, 2) Straight line Through *O* with positive slope In 1st quadrant only State or obtain algebraically that $y = 2$ Use suitable trigonometry Obtain correct answer a.e.f. decimals OK must be a complex number
7	(i) $a = -6$ (ii) $\mathbf{A}^{-1} = \frac{1}{a+6}\begin{pmatrix} 1 & -3 \\ 2 & a \end{pmatrix}$ $x = \frac{4}{a+6}, y = \frac{2-a}{a+6}$	M1 A1 B1 B1ft M1 A1ft A1ft	 2 5 **7**	Use det **A** = 0 Obtain correct answer Both diagonals correct Divide by det **A** Premultiply column by $\mathbf{A}^{-1}$, no other method Obtain correct answers from their $\mathbf{A}^{-1}$
8	(i) $u_2 = 4,\ u_3 = 9,\ u_4 = 16$ (ii) $u_n = n^2$ (iii)	M1 A1 B1 B1 M1 A1 A1	 2 1 4 **7**	Obtain next terms All terms correct Sensible conjecture made State that conjecture is true for $n = 1$ or 2 Find u_{n+1} in terms of n Obtain $(n + 1)^2$ Statement of Induction conclusion
9	(i) $\alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$ (ii) *Either* $\alpha + \beta = 5, \alpha\beta = 7$ $\alpha^3 + \beta^3 = 20$ $x^2 - 20x + 343 = 0$ *Or* $u^{\frac{2}{3}} - 5u^{\frac{1}{3}} + 7 = 0$ $u^3 - 20u + 343 = 0$	M1 A1 B1 B1 M1 A1 M1 A1ft M1 A1 M2 A2	 2 6 **8**	Correct binomial expansion seen Obtain given answer with no errors seen State or use correct values Find numeric value for $\alpha^3 + \beta^3$ Obtain correct answer Use new sum and product correctly in quadratic expression Obtain correct equation Substitute $x = u^{\frac{1}{3}}$ Obtain correct answer Complete method for removing fractional powers Obtain correct answer

10	(i)	M1 A1	 2	Attempt to combine 3 fractions Obtain given answer correctly
	(ii)	M1 A1 M1 A1 M1 A1	 6	Express at least first 3 terms using (i) All terms correct Express at least last 2 terms using (i) All terms correct in terms of n Show that correct terms cancel Obtain unsimplified correct answer
	(iii) $\frac{5}{2}$	B1ft	1	Obtain correct answer from their (ii)
	(iv) $\frac{2}{N+1}+\frac{1}{N+2}=\frac{7}{10}$	B1ft		Their (iii) – their (ii)
	$7N^2-9N-36=0$	M1		Attempt to clear fractions & solve equation, Obtain correct simplified equation
	$N=3$	A1 A1	 4 **13**	Obtain only the correct answer